CHAPTER 13 -- ELECTRIC FORCES and FIELDS

13.1) Coulomb's Law gives you the magnitude of the force on one point charge due to the presence of another point charge (there is an equal and opposite force on the "other" charge due to N.T.L.). Coulomb's Law states that that force equals $\left(\frac{1}{4\pi\varepsilon_o}\right)\frac{q_1q_2}{r^2}$, where the q's are the respective point charges involved, r is the distance between the charges, and the constant $1/(4\pi\varepsilon_o) = 9\text{x}10^9$ $\text{nt}\cdot\text{m}^2/\text{coulomb}^2$ (to save space, the symbol k will in some cases be used below in place of $1/(4\pi\varepsilon_o)$). Coulomb's Law does not give direction--that has to be eyeballed given the fact that unlike charges <u>attract</u> and like charges <u>repulse</u>.

a.) For this situation:

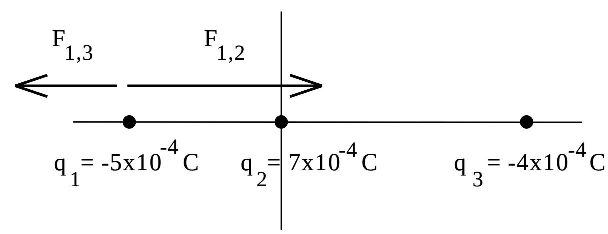


Figure I

$$F_{\text{net on 1}} = F_{\text{due to 2}} + F_{\text{due to 3}}$$
$$= kq_1q_2/r_{1,2}{}^2 - kq_1q_3/r_{1,3}{}^2.$$

Note that the negative sign is not due to the fact that q_1 and q_3 are like charges and, hence, repulse one another. Like charges do repulse one another, but in this case the negative sign relates the direction of the force on q_1 due to the presence of q_3. Switch the two and that direction would have been positive.

Pulling out q_1 and k, then rewriting, we get:

$$F_{net} = (9\text{x}10^9\ \text{nt}\cdot\text{m}^2/\text{C}^2)(5\text{x}10^{-4}\ \text{C})\left[(7\text{x}10^{-4}\ \text{C})/(3\ \text{m})^2 - (4\text{x}10^{-4}\ \text{C})/(8\ \text{m})^2\right]$$
$$= (321.88\ \text{nts})\ \mathbf{i}.$$

b.) Assuming the positive charge q_2 and the negative charge q_3 are fixed, q_1 would have to be placed to the right of the two. Why? Because only in that configuration will the repulsion of the closer, smaller negative charge q_3 be offset by the attraction to the larger, further positive charge q_2 (q_2 has been made large in the sketch to highlight its relative size). Mathematically, F_{net} will be:

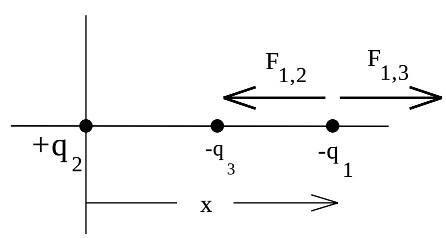


Figure II

$$-k(7x10^{-4}\ C)(5x10^{-4}\ C)/x^2 + k(4x10^{-4}\ C)(5x10^{-4}\ C)/(x - 5)^2 = 0$$

Using the Quadratic Formula, we get x = 20.49 meters.

Note: The Quadratic Formula yields a second solution--that of 2.85 meters. That solution has been thrown out because it places q_1 between q_2 and q_3.

c.) Again, for simplicity, assume that $k = (1/4\pi\varepsilon_o)$ for the calculations below. Also, the variable x is used to represent a magnitude in all three regions. That is, all negative signs will be unembedded.

There are three regions to be considered: the region x > 3, the region 0 < x < 3, and the region x < 0. (The variable x on each sketch identifies one possible point for that case.)

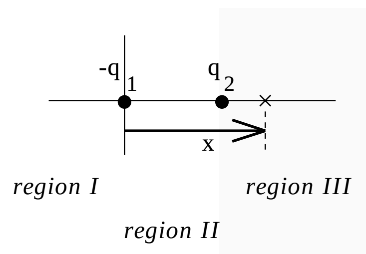
<u>For x > 3:</u>

$$E(x) = -kq_1/x^2 + kq_2/(x - 3)^2.$$

--as $x \to +\infty$, $E(x) \to 0$;

--as $x \to 3$, $E(x) \to +\infty$.

for x > 3
(i.e., in region III)



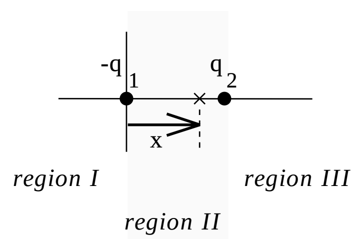
<u>For 0 < x < 3:</u>

$$E(x) = -kq_1/x^2 - kq_2/(3 - x)^2.$$

--as $x \to 3$, $E(x) \to -\infty$;

--as $x \to 0$, $E(x) \to -\infty$.

for 0 < x < 3
(i.e., in region II)



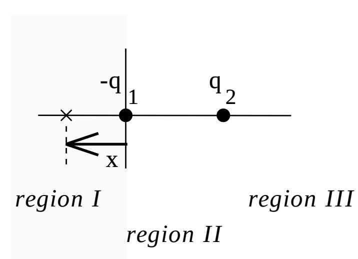
<u>For x < 0:</u>

$$E(x) = +kq_1/x^2 - kq_2/(x+3)^2.$$

--as $x \to 0$, $E(x) \to +\infty$;

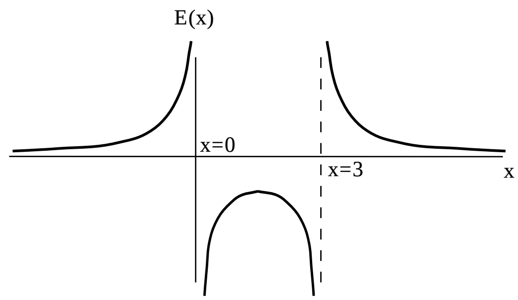
--as $x \to -\infty$, $E(x) \to 0$.

for x < 0
(i.e., in region I)

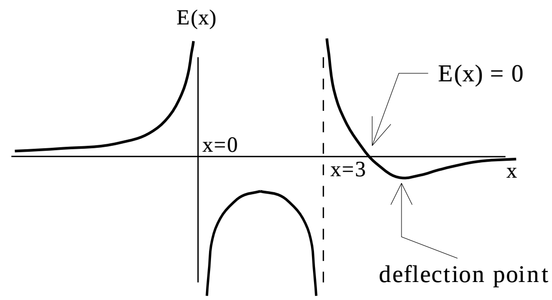


The graph of all of this information is shown on the next page. It is interesting to note that if the size of the charges had been unequal with, say, the magnitude of $q_1 > q_2$, there would have been a place to the right of q_2 where the electric field would have been zero. As E(x) would have still gone to zero at infinity, the graph would have had to have had a deflection point (a numerical value for that deflection point can be obtained by minimizing the electric field function associated with the region to the right of q_2--that is, by determining the x for which d[E(x)]/dx = 0). That situation is also shown below.

E(x) for equal charges:



E(x) for unequal charges:
(the smaller q is at x = 3)



13.2) Our $q = -5\text{x}10^{-8}$ coulombs, $m = 2\text{x}10^{-4}$ kg, and $F = 3\text{x}10^{-3}i$ nts.

a.) By definition, the magnitude of an electric field is:

$$\begin{aligned} E &= F/q \\ &= (3\text{x}10^{-3} \text{ nts})/(5\text{x}10^{-8} \text{ coulombs}) \\ &= 6\text{x}10^{4} \text{ nts/coulomb.} \end{aligned}$$

b.) When in an electric field, the acceleration of a negative charge will be opposite that of a positive charge. Positive charges accelerate in the direction of the electric field (that is how the electric field is defined). In other words, the negative charge's acceleration in the +i direction must be due to an electric field in the -i direction.

13.3)

a.) An electric field is a modified force field. When generated by a point charge, the field's magnitude is $\frac{1}{4\pi\varepsilon_o}\frac{Q}{r^2}$, where Q is the field-producing charge, r is the distance between the charge and point-of-interest, and $1/4\pi\varepsilon_o = 9\text{x}10^{9}\ \text{nt}\cdot\text{m}^2/\text{c}^2$ (to save space, we will call this constant k below).

Electric fields must be treated as vectors. When you have a number of point charges producing a net field at a point, you must be able to determine the <u>direction</u> of each field produced by each charge at the point.

In this problem, notice THERE IS NO CHARGE AT POINT (-a,-b). This is important. An electric field will exist in the vicinity of a field-producing charge whether other charges exist in the field or not.

Figure III to the right shows our problem, complete with vector components and the math needed to solve the problem. Of particular note is the fact that:

-- $\sin\theta = a/(a^2 + b^2)^{1/2}$ and

-- $\cos\theta = b/(a^2 + b^2)^{1/2}$.

With that in mind:

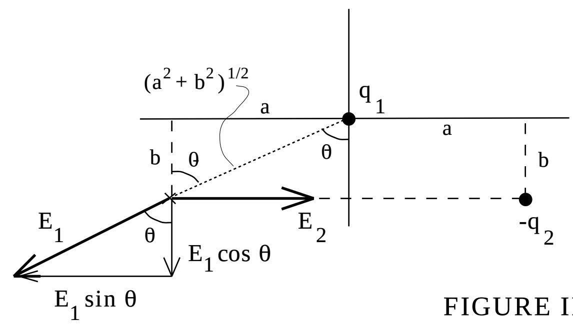


FIGURE III

$$
\begin{aligned}
E &= (-E_1 \sin\theta + E_2)\, i + (-E_1 \cos\theta) j \\
&= [-kq_1/((a^2 + b^2)^{1/2})^2][a/(a^2 + b^2)^{1/2}] + kq_2/(2a)^2]i + \\
&\qquad [-kq_1/((a^2 + b^2)^{1/2})^2][b/(a^2 + b^2)^{1/2}]\, j \\
&= [-kq_1 a/(a^2 + b^2)^{3/2} + kq_2/4a^2]i - [kq_1 b/(a^2 + b^2)^{3/2}]j.
\end{aligned}
$$

b.) With a = .4 meters, b = .3 meters, $q_1 = 7\ \mu C$ (i.e., $7\text{x}10^{-6}$ C), and $q_2 = 5\ \mu C$, we can pull out the k term and write:

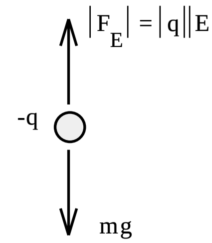
$$
\begin{aligned}
E &= 9\text{x}10^9 \{[-(7\text{x}10^{-6})[(.4)/(.4^2 + .3^2)^{3/2}] + 5\text{x}10^{-6}/[4(.4)^2]]i \\
&\qquad - (7\text{x}10^{-6}(.3)/(.4^2 + .3^2)^{3/2} j\} \\
&= (-131{,}287.5\ i - 151{,}200\ j)\ \text{nt/C}.
\end{aligned}
$$

Note: The numbers are a bit outrageous, but you get the idea.

c.) If you know the electric field intensity (as a vector) at a particular point, you can easily find the magnitude and direction of the force on ANY point charge placed in the field at that point using $F_{net} = qE$, where the q variable carries with it its sign (the direction of force on a negative charge will be opposite the direction of the electric field in which it resides). Doing so yields:

$$
\begin{aligned}
F_{net} &= (-.012\text{x}10^{-6}\ C)[(-131{,}287.5\ i - 151{,}200\ j)\ \text{nt/C}] \\
&= (1.575\text{x}10^{-3} i + 1.8\text{x}10^{-3}\ j)\ \text{nts}.
\end{aligned}
$$

13.4)



a.) A force on the charge must exist that opposes gravity, which means the electric force must be up. As the charge is negative, this means the electric field must be down.

b.) Using the math yields:

$$qE - mg = ma \qquad (= 0 \text{ as a is zero})$$
$$\Rightarrow \quad m = qE/g$$
$$= (-.04\ C)(-800\ nt/C)/(9.8\ m/s^2)$$
$$= 3.27\ kg.$$

Note that the <u>sign</u> of both the charge and the electric field is included in the equation $F = qE$.

13.5)

a.) This is a centripetal force problem. The center-seeking force on the circling electron is produced by its electric attraction to the proton in the nucleus (this is a Coulomb force). Using N.S.L. and $k = 1/4\pi\varepsilon_o$ we get:

$$F_{\text{due to proton}} = m_e a_{cent}$$
$$kq_e q_p/r^2 = m_e(v^2/r).$$

As the charge on a proton and an electron is the same (1.6×10^{-19} C), and as the radius of motion is half the diameter of the atom ($.5\times10^{-10}$ meters):

$$v = [kq^2/(m_e r)]^{1/2}$$
$$= [(9\times10^9\ nt\cdot m^2/c^2)(1.6\times10^{-19}\ C)^2/[(9.1\times10^{-31}\ kg)(.5\times10^{-10}\ m)]]^{1/2}$$
$$= 2.25\times10^6\ m/s.$$

b.) The force on the electron due to its presence in the proton's electric field is:

$$F_{\text{due to proton}} = kq_e q_p/r^2$$
$$= (9\times10^9\ nt\cdot m^2/c^2)(1.6\times10^{-19}\ C)(1.6\times10^{-19}\ C)/(.5\times10^{-10}\ m)^2$$
$$= 9.2\times10^{-8}\ nts.$$

The force on the electron due to its presence in the earth's gravitational field is:

$$F_{\text{due to gravity}} = m_e g$$
$$= (9.1\text{x}10^{-31}\text{ kg})(9.8\text{ m/s}^2)$$
$$= 8.9\text{x}10^{-30}\text{ nts.}$$

Comparing the two, we find that the electric force is:

$$F_{\text{electric}}/F_{\text{gravity}} = 9.2\text{x}10^{-8}/8.9\text{x}10^{-30}$$
$$= 1.03\text{x}10^{22}.$$

This means the electric force on an electron in an atom is ten-billion trillion times greater than the gravitational force on an electron close to the earth's surface.

13.6) The fact that the masses share a horizontal line means the electrical force on both masses will be in the horizontal.

a.) An f.b.d. for the forces acting on mass m_1 (= 2m) is shown to the right. N.S.L. yields:

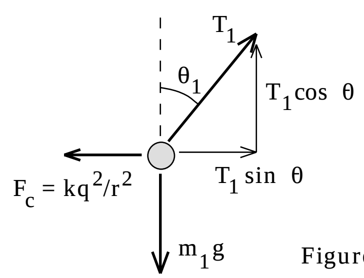


Figure IX

$\underline{\Sigma F_y:}$

$$T_1 \cos\theta_1 - m_1 g = m_1 a_y = 0$$
$$\Rightarrow \quad T_1 = m_1 g/\cos\theta_1.$$

$\underline{\Sigma F_x:}$

$$T_1 \sin\theta_1 - \frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2} = m_1 a_x = 0$$
$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2} = T_1 \sin\theta_1$$
$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2} = (m_1 g/\cos\theta_1)\sin\theta_1$$
$$= m_1 g \tan\theta_1.$$
$$= (2m) g \tan\theta_1.$$

Using a similar analysis on mass m_2, and noting that $m_2 = m$, we get:

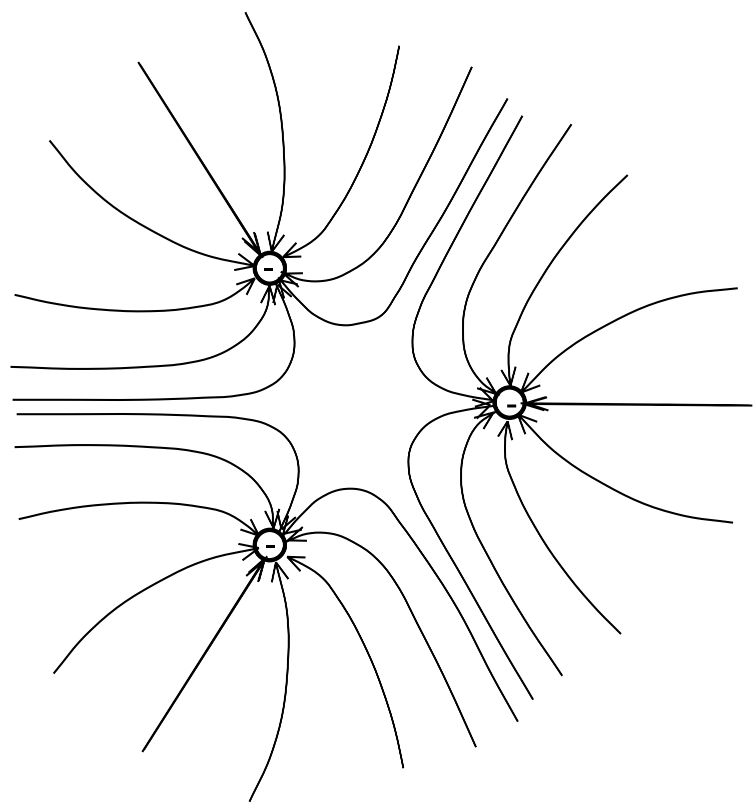
$$\frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2} = (m)g \tan\theta_2.$$

Equating the two $\frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2}$ terms yields:

$$2mg \tan\theta_1 = mg \tan\theta_2$$
$$\Rightarrow \quad \theta_2 = \tan^{-1}(2 \tan\theta_1)$$
$$= \tan^{-1}[2(\tan 35^o)]$$
$$= 54.5^o.$$

b.) With m = .03 kg and q = 5.5×10^{-10} coulombs, we can get r by going back to either of the final N.S.L. equations. Doing so yields:

$$\frac{1}{4\pi\varepsilon_o}\frac{q^2}{r^2} = 2mg \tan\theta_1$$
$$\Rightarrow \quad r = [(kq^2)/[2mg(\tan\theta_1)]]^{1/2}$$
$$= [(9\times10^9)(5.5\times10^{-10}\text{ C})^2/[2(.03\text{ kg})(9.8\text{ m/s}^2)(\tan 35^o)]]^{1/2}$$
$$= 8.13\times10^{-5}\text{ m}.$$



13.7) The electric field lines for the three negatively charged wires (cross section) are shown to the right. Note that their arrowheads point toward the negative charges. This makes sense--a positive test charge would be attracted to negative charges and, hence, would accelerate toward them if given the chance.

13.8)

a.) The direction of an electric field line at a point is defined as the direction a positive test charge would accelerate if put in the field at that point. In this case, that would be upward (i.e., in the +j direction).

b.) Positive charges accelerate in the direction of electric fields. Electrons do exactly the opposite. As the electric field in this case is upward, our electrons will accelerate downward.

c.) By determining the time of flight through the plates, we can determine the acceleration a_y needed if the electron is to just miss the plate's edge as it exits. Knowing that acceleration, we can use N.S.L. to determine the size of the force required to effect that motion and, from that, the size of the required electric field. Executing all that:

$$\Delta x = v_{1,x}\Delta t + .5a_x\Delta t^2.$$

As $a_x = 0$, $\Delta x = .12$ meters, and $v_{1,x} = 4\text{x}10^4$ m/s, we find the transit time to be:

$$\begin{aligned}\Delta t &= \Delta x/v_{1,x} \\ &= (.12 \text{ m})/(4\text{x}10^4 \text{ m/s}) \\ &= 3.0\text{x}10^{-6} \text{ seconds.}\end{aligned}$$

In that period of time, the y motion will be such that:

$$\Delta y = v_{1,y}\Delta t + .5a_y\Delta t^2.$$

As a_y is unknown (that is what we are looking for), and as Δy is downward, its value will be $\Delta y = -.02$ meters. Also, $v_{1,y} = 0$. With all of this, we find the acceleration to be:

$$\begin{aligned}a_y &= [\Delta y - v_{1,y}\Delta t] / [(1/2)\Delta t^2] \\ &= [(-.02) - 0] / [.5(3.0\text{x}10^{-6})^2] \\ &= -4.44\text{x}10^9 \text{ m/s}^2.\end{aligned}$$

The net force required for this motion will be:

$$\begin{aligned}F_{net} &= m_e a_y \\ &= (9.1\text{x}10^{-31} \text{ kg})(-4.44\text{x}10^9 \text{ m/s}^2) \\ &= -4.04\text{x}10^{-21} \text{ nts.}\end{aligned}$$

We haven't been given enough information to know how the plates are oriented relative to gravity, so we will do the problem first assuming gravity is a factor. Doing so yields a gravitational force downward coupled with an electric force on the electron that is also downward (remember, the charge on the plates produces an electric field upward, which means an electron will accelerate downward). Adding the two forces yields:

$$\begin{aligned} F_{net} &= -mg - F_e \\ &= -mg - q_eE \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad E &= (-mg - F_{net}) / q_e \\ &= [-(9.1\text{x}10^{-31}\text{ kg})(9.8\text{ m/s}^2) - (-4.04\text{x}10^{-21}\text{ nts})]/(1.6\text{x}10^{-19}\text{ C}) \\ &= 2.53\text{x}10^{-2}\text{ nt/C}. \end{aligned}$$

Ignoring gravity, we get:

$$\begin{aligned} F_{net} &= -F_e \\ &= -q_eE \end{aligned}$$

$$\begin{aligned} \Rightarrow \quad E &= -F_{net}/q_e \\ &= -[-4.04\text{x}10^{-21}\text{ nts}]/(1.6\text{x}10^{-19}\text{ C}) \\ &= 2.53\text{x}10^{-2}\text{ nt/C}. \end{aligned}$$

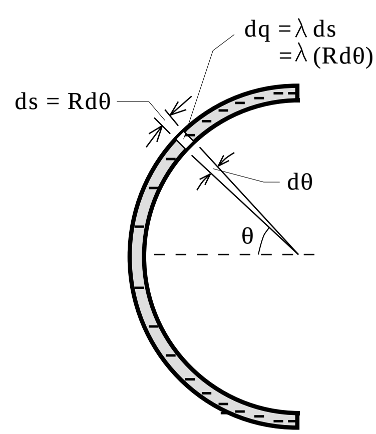
Evidently, in this case, it makes very little difference whether you include gravity in the calculation or not.

13.9)

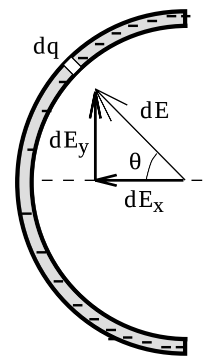
a.) As $\lambda = -k\theta$, the units for k must be such that when they are multiplied by radians, they yield the units of charge/length (these are the units for a linear charge density). As such, k's units must be coulombs per meter per radian, or C/(m·rad).

b.) Select an arbitrary, differential length ds on the rod at an angle θ with the horizontal. If ds is subtended by an angle $d\theta$, the arclength will equal $Rd\theta$, where R is the radius of the arc (see the figure to the right). Noting this, the charge magnitude on the differential section is:

$$dq = \lambda ds = (k\theta)(Rd\theta).$$



The electric field at the center of the arc generated by this differential charge is shown in the sketch to the right.



There will be a dq at an angle -θ whose electric field can be added to the electric field shown in the sketch. Exploiting symmetry, the y components of the two fields will add to zero. That means we only need to worry about the x components, or dE cos θ. Assuming positive x is to the left, we can write out $\int dE_x$ for the top charge, double it to include the bottom charge, and end up with:

$$
\begin{aligned}
E &= 2\int dE\cos\theta \\
&= 2\int_{\theta=0}^{\pi/2}\left[\frac{1}{4\pi\varepsilon_o}\frac{dq}{R^2}\right]\cos\theta \\
&= 2\int_{\theta=0}^{\pi/2}\left[\frac{1}{4\pi\varepsilon_o}\frac{\lambda ds}{R^2}\right]\cos\theta \\
&= 2\int_{\theta=0}^{\pi/2}\left[\frac{1}{4\pi\varepsilon_o}\frac{(k\theta)(Rd\theta)}{R^2}\right]\cos\theta.
\end{aligned}
$$

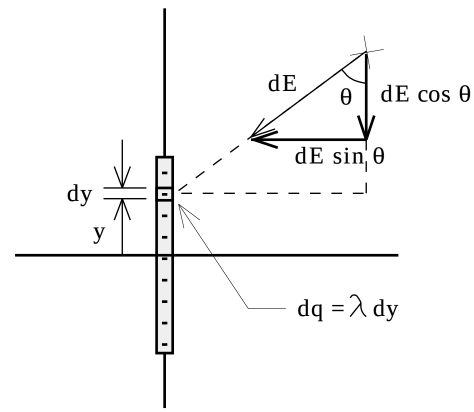
Simplifying where possible and integrating, we get:

$$
\begin{aligned}
E &= \frac{k}{2\pi\varepsilon_o R}\int_{\theta=0}^{\pi/2}(\theta\cos\theta)d\theta \\
&= \frac{k}{2\pi\varepsilon_o R}\left[\cos\theta+\theta\sin\theta\right]_{\theta=0}^{\pi/2} \\
&= \frac{k}{2\pi\varepsilon_o R}\left[\left[\cos(\pi/2)+(\pi/2)\sin(\pi/2)\right]-(\cos 0+0\sin 0)\right] \\
&= \frac{k}{2\pi\varepsilon_o R}\left[(\pi/2)-1\right].
\end{aligned}
$$

Note: Don't be put off by the θ's in the original expression. The integral was of the form $\int x\cos x\,dx$.

13.10) The sketch to the right shows the general setup. We need to determine the differential electric field dE at (L, L) due to an arbitrarily positioned differential charge dq on the rod. Having that field, we can break it into its components, then integrate to determine the net electric field in both the x-direction and y-direction. Using our sketch and doing the math, we get:

differential electric field at (L, L) due to arbitrary differential charge dq



$$dE = -dE \sin\theta\, i - dE \cos\theta\, j.$$

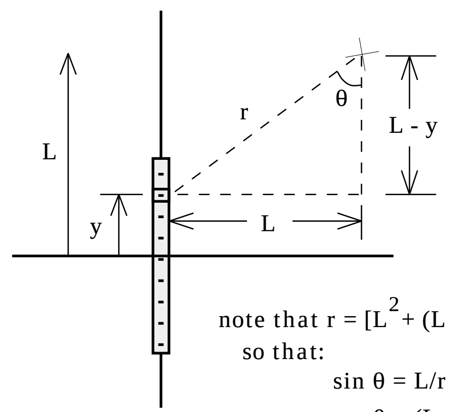
Integrating this gives us:

$$E = -\left[\int (dE\sin\theta)\mathbf{i} + \int (dE\cos\theta)\mathbf{j}\right].$$

To do this integral, we must either write dE in terms of $d\theta$, or we must write the $\sin\theta$, $\cos\theta$, and dE variables in terms of y's and dy. We will approach the problem by doing the latter.

Note 1: As was the case earlier, we don't necessarily need the angle θ. What we really need is the sine and cosine of θ. To get those quantities, consider the right triangle shown to the right.

determine the sine and cosine of θ



note that $r = [L^2 + (L - y)^2]^{1/2}$
so that:
$\sin\theta = L/r$
$\cos\theta = (L - y)/r$

Note 2: The differential electric field is due to the point charge dq. That means we can write the differential electric field as $dE = dq/(4\pi\varepsilon_o r^2)$. The magnitude of the differential charge is $dq = \lambda dy$, where λ is defined as the rod's charge/unit length, or Q/L in this problem. From the sketch, $\sin\theta = L/r$ and $\cos\theta = (L - y)/r$. Using all this information, we can proceed.

$$E = -\int (dE\sin\theta)\mathbf{i} - \int (dE\cos\theta)\mathbf{j}$$

$$= -\int\left(\frac{dq}{4\pi\varepsilon_o r^2}\sin\theta\right)\mathbf{i} - \int\left(\frac{dq}{4\pi\varepsilon_o r^2}\cos\theta\right)\mathbf{j}$$

$$= -\int\left(\frac{\lambda dy}{4\pi\varepsilon_o\left[\left(L^2+(L-y)^2\right)^{1/2}\right]^2}\left[\frac{L}{\left(L^2+(L-y)^2\right)^{1/2}}\right]\right)\mathbf{i}$$

$$- \int\left(\frac{\lambda dy}{4\pi\varepsilon_o\left[\left(L^2+(L-y)^2\right)^{1/2}\right]^2}\left[\frac{L-y}{\left(L^2+(L-y)^2\right)^{1/2}}\right]\right)\mathbf{j}$$

$$= -\int_{y=-L/2}^{L/2}\left(\frac{\lambda L\,dy}{4\pi\varepsilon_o\left(L^2+(L-y)^2\right)^{3/2}}\right)\mathbf{i} - \int_{y=-L/2}^{L/2}\left(\frac{\lambda(L-y)dy}{4\pi\varepsilon_o\left(L^2+(L-y)^2\right)^{3/2}}\right)\mathbf{j}$$

$$= \left[\frac{-\lambda L}{4\pi\varepsilon_o}\right]\left[\int_{y=-L/2}^{L/2}\left(\frac{dy}{\left(L^2+(L-y)^2\right)^{3/2}}\right)\mathbf{i} + \left[\int_{y=-L/2}^{L/2}\left(\frac{dy}{\left(L^2+(L-y)^2\right)^{3/2}}\right) + \int_{y=-L/2}^{L/2}\left(\frac{-y\,dy}{L\left(L^2+(L-y)^2\right)^{3/2}}\right)\right]\mathbf{j}\right]$$

What is important here is the setup of the integrals. For those gung-ho souls who would like an answer, the integrals can be evaluated by noting that the denominator is the square root of a quadratic cubed, and by using the following relationships:

$$\int\frac{dy}{\left(ay^2+by+c\right)^{3/2}} = \frac{4ay+2b}{\left(4ac-b^2\right)\left(ay^2+by+c\right)^{1/2}}$$

and

$$\int\frac{y\,dy}{\left(ay^2+by+c\right)^{3/2}} = -\frac{2by+4c}{\left(4ac-b^2\right)\left(ay^2+by+c\right)^{1/2}}.$$

Note: No, I didn't divine (sic) these relationships. They are available in any Table of Integrals.

13.11) If we can show that the motion is simple harmonic in nature, we can determine the frequency of the oscillation. With the positive charge on the positive side of the loop, the direction of the electric force will be negative and N.S.L. yields:

$\underline{\sum F_x:}$

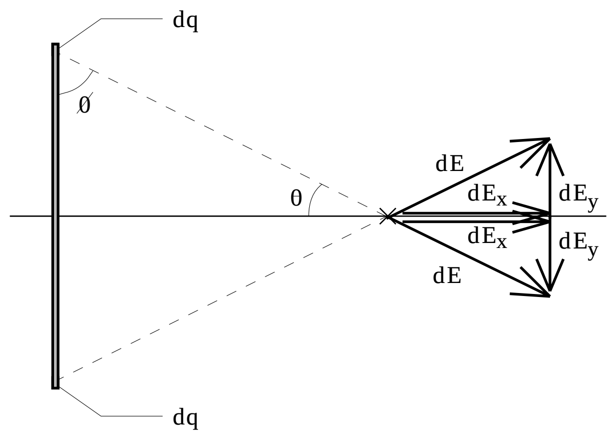
$$-qE = ma$$

$$\Rightarrow \quad a + (qE/m) = 0.$$

If we didn't have a function for the electric field generated by a hoop down its axis, we would have to derive that expression. Fortunately, it was derived in section E-2 of Chapter 13 (the derivation was for positive charge--the magnitude of E will be the same). In the original derivation, a substitution was made for a sin θ term. Before that substitution was made, the derived expression for the magnitude of E looked like:

$$\mathbf{E}(x) = \left[\frac{Q}{4\pi\varepsilon_o\left(R^2 + x^2\right)}\right]\cos\theta\ \mathbf{i}$$

$$= \left[\frac{Q}{4\pi\varepsilon_o\left(R^2 + x^2\right)}\right]\sin\phi\ \mathbf{i},$$



where $\phi = (90^o - \theta)$ . . . see the sketch to the right. Using this relationship in the x-direction, N.S.L. yields:

$$a + \left(\frac{q}{m}\right)[E] = 0$$

$$\Rightarrow \quad a + \left(\frac{q}{m}\right)\left[\frac{Q}{4\pi\varepsilon_o\left(R^2 + x^2\right)}\right]\sin\phi = 0$$

$$\Rightarrow \quad a + \left[\frac{qQ}{4\pi\varepsilon_o m\left(R^2 + x^2\right)}\right]\sin\phi = 0$$

This is not the characteristic equation for simple harmonic motion. In the first place, the acceleration a is a translational term while the displacement ϕ is an ANGULAR displacement term. They have to be of the same ilk for this technique to cooperate. Also, there is an x variable in the denominator of the "constant."

If you hadn't noticed any of this, you may be tempted to manipulate the expression as though all was well. In that case, the small angle approximation would have allowed the sine to become the angle itself (measured in radians), and you would have had ended up with an expression of the form acceleration plus a constant times a displacement term equals zero (again, this assume you hadn't noticed that the constant wasn't a constant but had the variable x in it--for the sake of argument, let's assume you called that variable b). Once in that form, you would have known that the square root of the constant was equal the angular frequency of the motion. Following that logic, the angular frequency would have been:

$$\omega = \left(\frac{qQ}{4\pi\varepsilon_o m\left(R^2 + b^2\right)} \right)^{1/2}$$

and the frequency $\nu = \dfrac{\omega}{2\pi}$ would have been:

$$\nu = \frac{\left(\dfrac{qQ}{4\pi\varepsilon_o m\left(R^2 + b^2\right)} \right)^{1/2}}{2\pi}.$$

Unfortunately, NONE OF THIS WILL WORK! The proton does NOT oscillate with simple harmonic motion and, as such, we have no easy way of determining the frequency of the motion.